Flow Visualizations Method for Flow Past a Flat Plate

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Abstract

We study one of the external flows that past a plate to include various angles of attack. The problem was solved using elliptic coordinate. The problem of running through a plate with different angles is calculated. The plate in this article is not just an ordinary plate, but a plate obtained from a perspective that makes it easy to see difficult streams. The streamlines are drawn. Lift and drag are analyzed from the streams of the resulting images.

Key words: potential flow, conformal transformation

INTRODUCTION

The present paper is to carry out experiments on boundary layers. Such experiments are needed to verify the positive effect that is inflicted by techniques to manipulate the boundary layer. In practice, it is still difficult to measure the velocity profiles within the boundary layer. The present study will compare results from the theory of boundary layers with the results from experiments in the simplest setting; a flat plate at zero degrees of incidence at modest Reynolds numbers. The measurements will be compared with the relations from theory to assess the accuracy at the measurements. In short, the goal of this study is to measure the velocity profile in the boundary layer of the flat plate and compare the results with the results from theory.

Irrotational flow patterns around body of flap plate shape are the subject of this paper. It will be assumed that the fluid is inviscid and incompressible and the motion 2−dimensional. The 2−dimensional incompressible continuity equation guarantees the existence of a stream functionψ, from which the velocity components can be derived as

$$u = -\frac{\partial\psi}{\partial y}\ , \quad v = \frac{\partial\psi}{\partial x}\ .$$

For irrotational motion, the stream function satisfies Laplace’s equation: $\nabla^2\psi = 0$. Likewise, the condition of irrotationality guarantees the existence of another scalar function ϕ, called the velocity potential, which is related the velocity components by

$$u = -\frac{\partial\phi}{\partial x}\ , \ v = -\frac{\partial\phi}{\partial y}\ .$$

And potential function satisfies Laplace’s equation : $\nabla^2\phi = 0$. We obtain $\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y}$, $-\frac{\partial\phi}{\partial y} = \frac{\partial\psi}{\partial x}$.

These are Cauchy−Riemann condition for $\phi(x,y)+ i\psi(x,y)$ to be analytic function of the complex variable x+iy = z and so we can put

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$$\phi(x,y)+ i\psi(x,y) = f(x+iy) = f(z),$$

where f is analytic function of z (Chorlton, 1967).

These facts can be useful for analyzing 2–dimensional potential flow for certain kinds of the boundary conditions. If we can find a solution of Laplace's equation for a simple boundary in z–plane, we can apply any analytic mapping we choose, and map the boundary and streamlines to another complex plane. Since the streamlines conformed to the boundary in the original plane, they automatically conform to the transformed boundary in the transformed plane, and since the mapping is analytic, the transformed streamlines are solution of Laplace's equation, just as were the streamlines in the original plane. Likewise the velocity potential maps from the original to the transformed plane.

Elliptic coordinates

If we look at that study from a different perspective, we can get a flat flow ahead of time. These are the various images that can be obtained by imaging using one technology. Further analysis reveals the lift and drag forces. Here we will use the properties of flow past a circle. In studying the streams, the mathematical terms of the streams are summarized as follows. The mapping of the plane $z = x + iy$ on the plane $\zeta= \xi+i\eta$ given by the relation

$$z = c \cosh\zeta = c \cosh (\xi+i\eta) \qquad \text{-----(1)}$$

which may also be written as

$$(x + iy) = \frac{c}{2}(e^{\xi+i\eta} + e^{-\xi-i\eta}).$$

$$x + iy = c(\cosh\xi \cos\eta + i \sinh\xi \sin\eta).$$

Separation of real and imaginary parts on both sides gives now

$$x = c \cosh\xi \cos \eta, \quad y = c \sinh \xi \sin \eta.$$

η can be eliminated by the use of $\cos^2\eta + \sin^2\eta = 1$, resulting in

$$\frac{x^2}{c^2\cosh^2\xi}+\frac{y^2}{c^2\sinh^2\xi}=1. \qquad \text{-----(2)}$$

The elimination of ξ by the use of $\cosh^2\xi - \sinh^2\xi = 1$, resulting in

$$\frac{x^2}{c^2\cos^2\eta}-\frac{y^2}{c^2\sin^2\eta}=1. \qquad \text{-----(3)}$$

For constant ξ, (2) represents an ellipse with the semi–axis

$$a = c \cosh\xi \;, \; b = c \sinh\xi.$$

Its eccentricity is given by

$$\sqrt{a^2+b^2}=c \qquad \text{-----(4)}$$

and thus independent of ξ. Thus, if ξ varies, one obtains a system of confocal ellipses with foci $x = \pm c$, $y = o$.

On the other hand, for constant η, (3) represents a hyperbola with a semi–axis

$$a = c \cos\eta, \; b = c \sin \eta. \qquad \text{-----(5)}$$

Therefore independent of η and equal to that of ellipses given in (4). With variable parameter η, (3) represents a system of hyperbolas, confocal among themselves and with the system of ellipses (Sommerfeld,1950).

It is of interest to consider the limiting parameter values $\xi = 0$ and $\xi = \infty$ in the family of ellipses: the value $\xi = 0$ gives by (2) the focal line

$$y = 0, \quad -c < x < c$$

and the value $\xi = \infty$ gives an infinity extended ellipse.

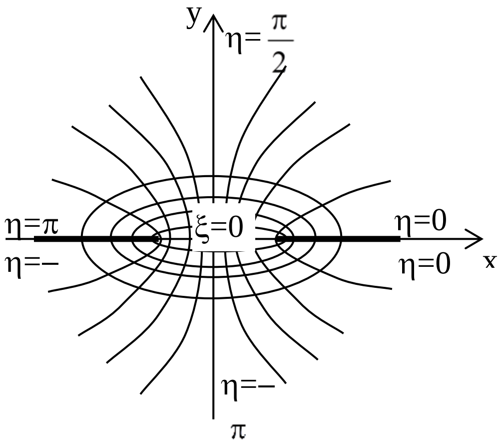


Fig-1 The system of confocal ellipses and hyperbolas ξ=constant and η= constant.

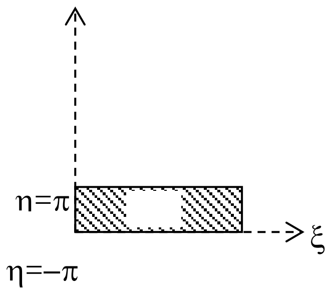


Fig-2 The mapping of the z–plane on the ζ–plane.

As regards the parameter η, note the particular values

$$\eta = 0, \quad \eta = \pm\frac{\pi}{2}, \quad \eta = \pm\pi.$$

The value of $\eta = 0$ characterizes by (3), the part of the real axis to the right of c: $y = 0$, $x > c$. $\eta = \pm\frac{\pi}{2}$ corresponds to $x=0$, the hyperbola coincides with the y–axis. $\eta = \pm\pi$ gives the part of the x–axis to the left of c: $y = 0, \quad x < -c$.

In order to obtain a unique relation between the coordinates ξ, η and the points of the x, y–plane, ξ and η must be restricted to a certain domain.

One way of doing this is to assign the values $0 < \eta < \pi$ to the upper halves of the hyperbolas, and the values $-\pi < \eta < 0$ to the lower halves. By this rule the entire x, y–plane is mapped on the strip to the ξ,η–plane

$$0 < \xi < +\infty, \qquad -\pi < \eta < \pi$$

indicated in above Fig-2.

Note that the one to one correspondence between the x, y–plane and the ξ,η–strip does not include the boundary of the strip; either the upper or the lower half of the borderline must be omitted. Or one might also consider boundary points that are mapped on the same point in the x, y–plane as identical (viz.0, $\pm\eta$; $\xi\neq 0$, $\pm\pi$).(Sommerfeld,1950)

The problems arising from the flow past a plate will be treated by means of elliptic coordinates. We shall use a complex potential of the form

$$\phi + i\psi = f(\zeta), \text{ where } f(\zeta) = \text{const} \ \sinh\xi \qquad \text{———————}(6)$$

Here, ζ is through to be related to z according to (1), so that $f(\zeta)$ becomes a complex function of z.

Flow past a vertical flat plate

The connection of elliptical coordinates ξ, η with Cartesian coordinates x, y is given by

$$x+iy = c\cosh(\xi+i\eta). \qquad \text{———————(7)}$$

Here c is half the focal distance of Fig-2 and, at the same time, half the length of the projection of our plate in the x,y–plane; the plate is supposed to be infinite in ± z–direction.

In elliptic coordinates the front and back sides of the plate are simultaneously given by

$$\xi = 0, \quad -\pi < \eta < \pi,$$

corresponding to the infinitely narrow ellipse of Fig-2. For these values the right member of (1) is real, hence y=0 and $-c < x < c$.

On the other hand, for

$$\eta = \pm\frac{\pi}{2}, \quad 0 < \xi < \infty$$

the right member of (7) becomes a positive or negative purely imaginary number, hence x = 0 , y > 0 and x= 0,y < 0, as seen in Fig.2.

Consider now the analytic function

$$\phi+i\psi = \text{const}\,\sinh(\xi+i\eta). \qquad \text{—————————(8)}$$

Let const be equal to iqc and c > 0, where q is a real quantity having the dimension of a velocity.

Now, $\phi+i\psi$ is not only an analytic function of $\xi+i\eta$, but through (1) be analytic function of (x+iy) likewise. Consequently ϕ and ψ may be interpreted as velocity potential and stream function (5). They satisfy such boundary conditions in the x, y–plane as are required by the problem under consideration. The values of ϕ and ψ along certain lines of the x, y–plane have been listed in the following table, the auxiliary variables ξ and η playing the role of parameters:

Table-1 Uniform flow

ξ,η–plane	x , y - plane	values of ϕ, ψ
$\xi=0$	$x=c\cos\eta$, $y = 0$	$\psi=0$, $\phi=-qc\sin\eta$
$\eta=-\frac{\pi}{2}$	$x=0$, $y=-c\sinh\xi<0$	$\psi=0$, $\phi=qc\cosh\xi>0$
$\eta=+\frac{\pi}{2}$	$x=0$, $y=c\sinh\xi>0$	$\psi=0$, $\phi=-qc\cosh\xi<0$
$\xi\to\infty$	$x^2+y^2=\frac{c^2}{4}e^{2\xi}\to\infty$	$\psi=qx$, $\phi=-qy$

The first three lines of this table are explained in our previous remarks, if one takes into account that the right member of (8) becomes real for $\xi = 0$ as well as for $\eta = \pm\frac{\pi}{2}$. As regards the last line, the effect of the limit $\xi\to\infty$ on x, y, etc. can be easily seen if one separates real and imaginary parts in (7) and (8).

For large ξ, $\quad x+iy = \frac{c}{2}e^{\xi+i\eta} = \frac{c}{2}e^{\xi}(\cos\eta + i\sin\eta).$

Then $x = \frac{c}{2} e^{\xi} \cos\eta, \quad y = \frac{c}{2} e^{\xi} \sin\eta$

and $\phi = -q \frac{c}{2} e^{\xi} \sin\eta, \quad \psi = q \frac{c}{2} e^{\xi} \cos\eta$ ———————(9)

in agreement with the last line of the table.

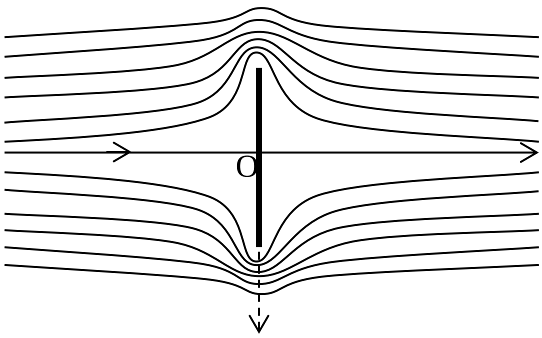


Fig-3 Flow past a vertical flat plate

Flow Visualization by the Transformation

In this section we will study the flow over the flat plate using visualization method. The flow of streamline must be resolved. By physical meaning of flow, flow property is satisfied. The flow on the plate will be analyzed using the existing circular properties. Then the circle will be transformed into a flat according to the transformation.

The transformation $z = f(Z) = Z + \frac{a^2}{Z}$, a real, is one of the simplest and most important transformation of two dimensional motions. By means of this transformation we can map the Z–plane on the z–plane, and vice versa. Since $f'(Z) = 1 - \frac{a^2}{Z^2}$, the mapping is conformal except at $Z = \pm a$ (F Chorlton, 1967). Then we will calculate the liquid properties. The flow from the circle to the plate will be changed without compromising the flow properties. The boundaries of the fluid in one plane, being streamlines, transform into the boundaries of the fluid in the other plane. Sources, sinks, and vortices in one plane map into sources, sinks, and vortices of the same strength in the other plane under a conformal transformation. Since the two flow patterns are not the same, the velocities are not mapped one to one, but they are proportional to one another, depending on the mapping function.

Flow Past a Flat Plate

We now consider the image of the circle $Z = ae^{i\theta}$, with $z = Z + \frac{a^2}{Z}$.

Uniform flow past the flat plate, parallel to it is given by w = Uz. Therefore complex potential for corresponding flow past a cylinder is $w = U\left(Z + \frac{a^2}{Z} \right)$.

Then we get the components $x + iy = ae^{i\theta}$□□□ $+ \frac{a^2}{a e^{i\theta}} = 2a\cos\theta = 2X$.

Hence the circle is mapped to a straight line y = 0, –2a < x < 2a, with the exterior of the circle mapping to the exterior of the line section (–2a, 2a).

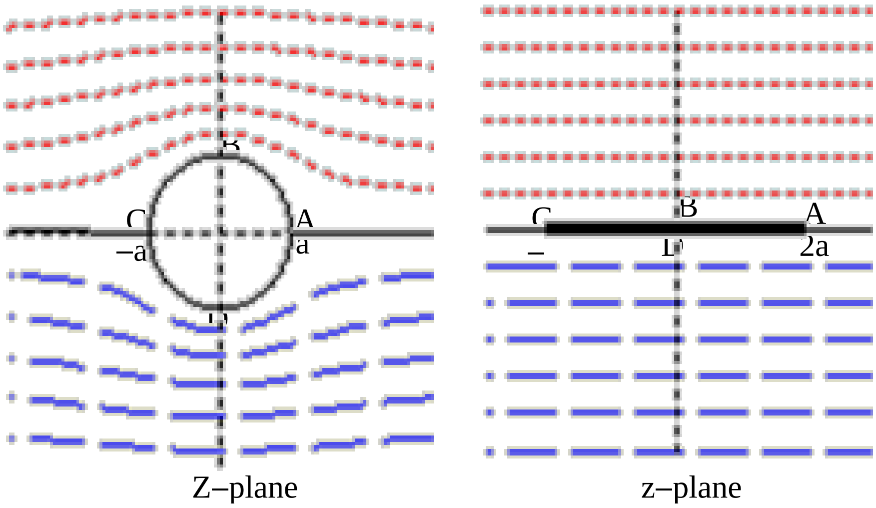


Fig-4 Flow past a horizontal flat plate

We see that the first contour r = a corresponds to the streamline and second contour the flat plat length 4a is also streamline. The two flow patterns are symmetric. This symmetry appears when the flow direction is parallel with the axis of symmetry of the body. The stagnation points are coincided with the ends of each body. Since velocity is the same at the upper and lower surface, the pressure is the same. The nature of pressure distribution supports the complex symmetry observed in streamlines seen above while showing that upper and lower surface distributions cancel each other out. So that effect gives non lifting flow. The streamline contours of the transformation planes are the same. Thus we calculate difficult flow patterns.

Flow Past an Oblique Plate

Consider a flat plate in the z–plane , y = 0 and –2a < x < 2a and far from the plate, the flow is uniform magnitude U, at angle θ to the x–axis. Therefore complex potential becomes w $= U\left(Ze^{-i\theta} + \frac{a^2}{Z} e^{i\theta}\right)$, θ= 90°. Then we get the following symmetric streamlines. The velocity, pressure are balance at the upper and lower sides. It has no lift.

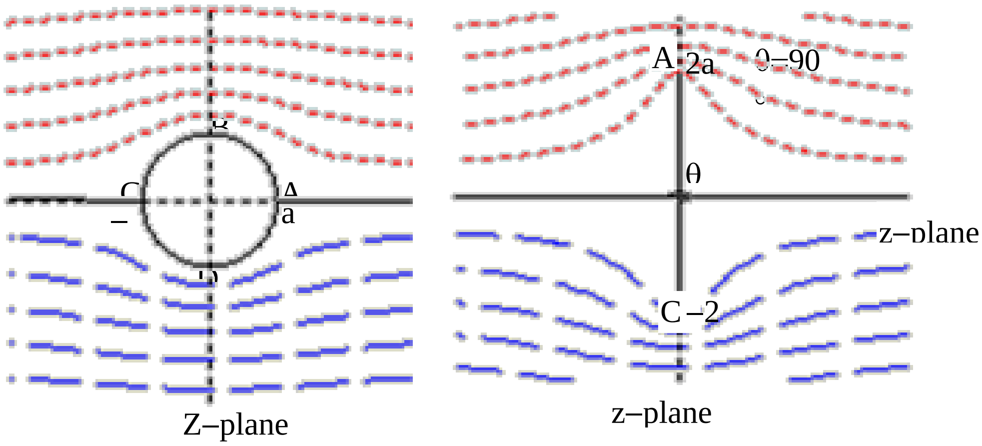
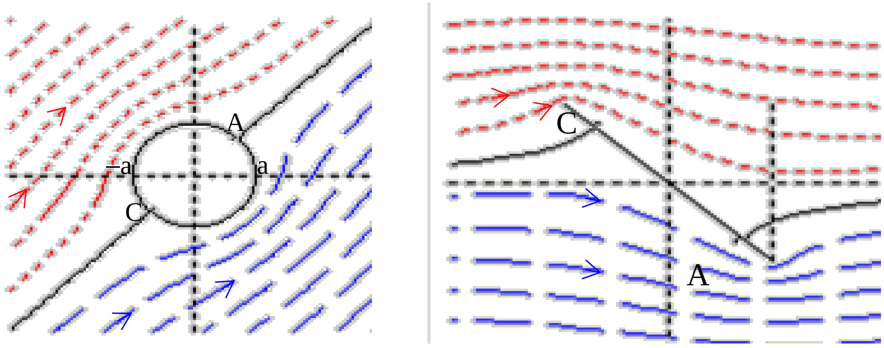


Fig-5 Flow past a vertical flat plate

Writing $z = Ze^{-i\theta} + \frac{a^2}{Z} e^{i\theta}$, the line in the z–plane is mapped to a circle in the Z–plane. Also, as $|z| \to \infty$, $z \to Z$, so the flow far from the body that is the same in the z–plane and Z–planes. Consider flow angle θ to a circle.



Z–plane z–plane

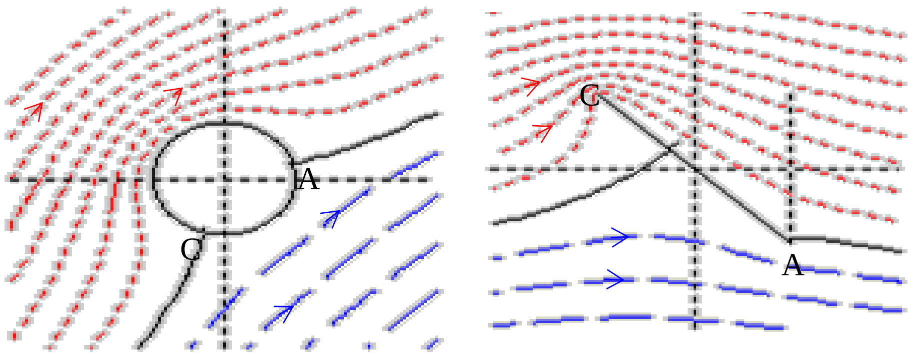
Fig-6 Flow past an oblique flat plate

We also get the above symmetric streamlines. The velocity, pressure are balance at the upper and lower sides. There is no lift.

Flow Past an Oblique Plate with Circulation

The same analysis can be repeated but now including circulation k about the body. The complex potential becomes

$$w = U\left(Ze^{-i\theta} + \frac{a^2}{Z} e^{i\theta}\right) - \frac{ik}{2\pi} \log z\ .$$



Z–plane z–plane

Fig-7 Flow past an oblique flat plate with circulation

If we add this circulation to the flow, rare stagnation point falls into the trailing edge. In this way, we have no longer an infinite velocity on the trailing edge as it is illustrated. If the circulation $|\kappa|/4\pi Ua$ is greater than that `1′, the leading stagnation point moves to lower surface.

The streamline patterns do not have not the same velocity at the upper sides and the lower sides. Thus the pressures are not the same. It produces lift.

RESULTS AND CONCLUSION

In this paper we have analyzed and interpreted the fluid flows by using visualization technique. Building up a streamline, to interpret fluid flow is a difficult and also time-consuming process. Solving the difficulties related to fluid flow with technical assistance is the main task of this paper. First of

all, consistency of the fluid flow with technical process (our software) used is examined and interpretation is then proceeded. To solve fluid flow problems with complex vector pattern is very difficult and time consuming. With technical assistance (i.e. with Matlab program) complex vector patterns were easily visualized. Resulted patterns were interpreted from different sides of view. Fluid dynamics is aimed at predicting the velocity and pressure fields in the flows past rigid bodies. Of particular interest are the velocity and pressure distributions on the body surface. External flows called flows past rigid bodies can also be easily visualized by technical assistance. Being integrated with the visualization of external flows, we can figure out whether it can create lift or not. On the other hand, the velocity and pressure form the figure that gives the aerodynamic force acting upon the body between the flow and the body.

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